AN OVERVIEW OF THE FUZZY SETS ON PROCESS CAPABILITY ANALYSIS

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ABSTRACT

Process capability analysis (PCA) is the ability of a process that measures the degree of providing customer expectations. Generally, the expectations of customer have been defined by using specification limits. A process capability index (PCI) summarizes ability of process to conform specification limits. Generally, this comparison is made by analyzing ratio of specification limits and six-sigma limits. After the inception of the fuzzy set theory (FST) by Zadeh in 1965, PCA has been greatly influenced at a significant level. It is possible to see many papers about usage of the FST on PCA. In this paper, the effects of the FST on PCA have been analyzed and the recent trends have been summarized. By the way, a roadmap about possible future directions of the FST has been suggested.

Keywords: Process capability analysis, process capability index, the fuzzy set theory, six sigma

1. INTRODUCTION

Process capability analysis (PCA) can be broadly defined as the ability of a process to meet customer expectations. The main outputs of PCA are process capability indices (PCIs) that provide a numerical measure of whether the process is capable or not. PCIs are summary statistics which measure the actual or the potential performance of the process characteristics relative to the target and specification limits. A process capability index (PCI) is a number that summarizes the behavior of a product or process characteristic relative to specifications. Generally, this comparison is made by forming the ratio of the width between the process specification limits to the width of the natural tolerance limits [1-3]. This has provided more information and more sensitiveness on PCIs. The fuzzy set theory (FST) that provides a simple way to reason with vague, ambiguous, and imprecise can be used to increase sensitiveness, flexibility and usability of PCIs. It is easy for analyzing usage of the FST in PCA at literature. In this paper, a literature review based on published papers on PCIs and the FST is analyzed. For this aim, some analyzes with respect to types of PCIs, numbers and types of the papers, years and journals has been summarized. We aimed to show a trend about the usage of the FST in PCIs and will give some directions on future research of the fuzzy PCIs. The rest of this paper has been organized as follows: PCA and indices are briefly introduced into Section 2. A comparative literature analysis is shown into Section 3. The obtained results and future research directions are summarized into Section 4.

2. PROCESS CAPABILITY AND PROCESS CAPABILITY INDICES

Process capability indices (PCIs), which provide numerical measures on whether a process meets the customer requirements or not, have been popularly applied for evaluating process performance. They are summary statistics which measure the actual or potential performance of the process characteristics relative to the target and specification limits (SLs). Several PCIs such as C_p, C_{pk}, C_{pm}, and C_{pmk} are used to estimate the capability of a process [4].

2.1 The Index C_p

C_p is defined as the ratio of specification width over the process spread. The specification width represents customer and/or product requirements. The process variations are represented by the specification width. If the process variation is very large, the C_p value is small and it represents a low process capability [5]:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad (1)$$

where σ is the standard deviation of the process, USL and LSL represent the upper and lower specification limits, respectively.

2.2. The Index C_{pk}

C_p indicates how well the process fits between upper and lower specification limits. It never considers any process shift and simply measures the spread of the specifications relative to the six-sigma spread in the process. If the process average is not centered near the midpoint of specifications limits (*m*), the C_p index gives misleading results. Therefore Kane [6] introduced C_{pk} which is used to provide an indication of the variability associated with a process [5]:

$$C_{pk} = \min\left\{C_{pl}, C_{pu}\right\} = \frac{\min\{USL - \mu, \mu - LSL\}}{3\sigma} \quad (2)$$

2.3. The Index C_{pm}

A well-known pioneer in the quality control, G. Taguchi, pays special attention on the loss in product's worth when one of product's characteristics deviates from the customers' ideal value *T* (target value). To take this factor into account, Hsiang and Taguchi introduced the index C_{pm} [7]:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}} \quad (3)$$

2.4. The Index C_{pmk}

Pearn et al. [8] proposed the index C_{pmk}, which combines the features of the three earlier indices C_p, C_{pk} and C_{pm}. The index C_{pmk} alerts the user whenever the process variance increases and/or the process mean deviates from its *T*. [9]:

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\} \quad (4)$$

3. LITERATURE REVIEW OF THE FUZZY SETS ON PROCESS CAPABILITY ANALYSIS

The FST have been introduced by Zadeh [10] as an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition an element either belongs or does not belong to the set. By contrast, the FST permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1]. After the inception of the FST, some studies have been made to combine PCIs and the FST. In this section, some of these studies are briefly summarized as following: Yongting [11] defined a formula of index C_{pk} to measure fuzzy quality. Lee et al. [12] presented a model for designing process tolerances to maximize the PCI. Lee [13] proposed a model to calculate the fuzzy PCI when observations were fuzzy numbers. Chen et al. [14] proposed a method to incorporate the fuzzy inference with the PCI in the bigger-the-best type quality characteristics assessments. Chen et al. [15] proposed a fuzzy inference method to select the best among the competing suppliers. Gao and Huang [16] emphasized that process tolerances had influences not only on manufacturing costs, but also on the achievement of the required specifications of a product. Parchami et al. [17] introduced new PCIs in triangular fuzzy numbers. Parchami et al. [18] obtained a $(1-\alpha)100\%$ fuzzy confidence interval for fuzzy PCIs. Tsai and Chen [19] extended the application of the index C_p in the manufacturing industry to a fuzzy environment. Kaya and Kahraman [20] proposed a methodology based on PCIs to control air pollution. Parchami and Mashinchi [21] applied Buckley's estimation approach to several PCIs. Kahraman and Kaya (2008) proposed fuzzy PCIs to control the pH value of dam's water for agriculture. Kaya and Kahraman [22] applied the fuzzy process capability analyses when the specifications limits were triangular fuzzy numbers. Kaya and Kahraman [23] analyzed the risk assessment of air pollution in Istanbul by using fuzzy PCIs. Kahraman and Kaya [24] used PCIs to risk assessment of drought effects. Kaya and Kahraman [25] proposed a methodology based on PCIs to prevent air pollution. Chen and Chen [26] presented a method to incorporate fuzzy inference with process capability. Hsu and Shu [27] presented a method combining the vector of fuzzy numbers to produce the membership function of fuzzy estimator of the index C_{pm}. Wu [28] presented a set of confidence intervals that produces triangular fuzzy numbers for the estimation of C_{pk} index. Kaya (2014) analyzed the process incapability index under fuzziness. Kahraman et al. [41] analyzed PCIs under intiutionistic fuzzy sets. Geramian et al. [42] integrated quality loss function and PCIs for continuous improvements. Parchami et al. [43] analyzed two fuzzy PCIs, C_p and C_{pm}, under fuzzy hypothesis test and fuzzy p-value.

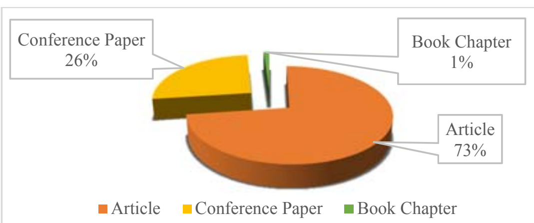

Figure 1. The percentages of papers that contains the FST and PCIs

The percentages of document types are shown in Figure 1. As seen easily, the percentage of articles is the highest value with the percentage of 73. The number of articles that includes fuzzy PCIs is 67. The numbers of conference papers and book chapters are 24 and 1, respectively.

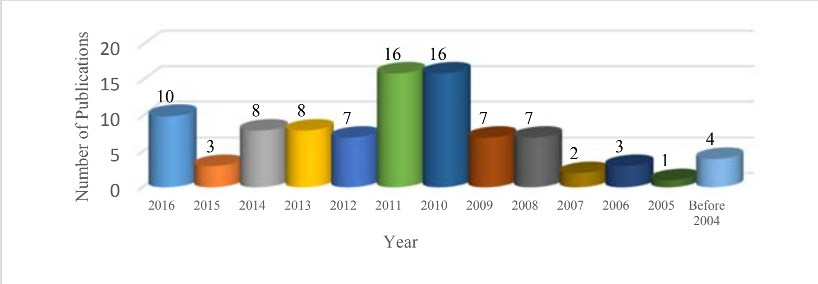

Figure 2. The numbers of the papers that contains the FST and PCIs based on years

A graphical summary about the numbers of the papers is shown in Figure 2. We can see that in 2010 and 2011, the number of the papers reached the highest value. In 2016, the new extensions of the FST has been starting to use and the number of papers has been achieved the value of 10.

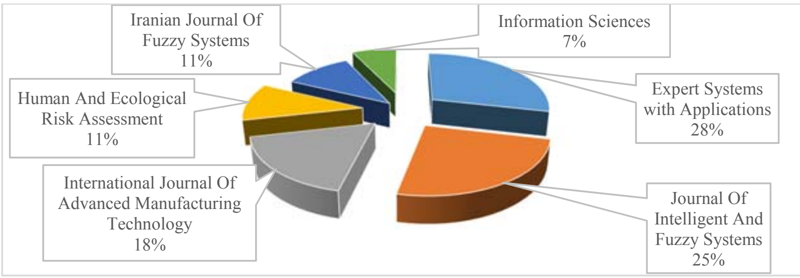

Figure 3 The percentages of papers based on journals

The percentages of articles with respect to journals is given in Figure 3. The studies are the most interested journal is Expert Systems with Applications. It has published the highest number of articles on fuzzy PCIs. Then the Journal of Intelligent and Fuzzy Systems follow it. A classification of the papers with respect to fuzzy PCIs is summarized in Table 1. According to Table 1, the PCIs C_p and C_{pk} are the two mostly used indices with FST. In recent years, some new indices about incapability and accuracy of process has been starting to analyze under fuzzy environments.

Table 1. A classification of the papers with respect to capability indices

	Fuzzy PCIs				
	$\tilde{C}_p$	$\tilde{C}_{pk}$	$\tilde{C}_{pm}$	$\tilde{C}_{pmk}$	**Others**
Yongting [11]	√	√			
Lee et al. [12]	√				
Lee [13]		√			
Chen et al. [14]	√				√
Chen et al. [15]			√		
Gao and Huang [16]					√
Parchami et al. [17, 18]	√	√	√	√	
Parchami and Mashinchi [21]	√	√			
Tsai and Chen [19]	√				
Kaya and Kahraman [23]	√	√	√		

	Fuzzy PCIs				
	$\tilde{C}_p$	$\tilde{C}_{pk}$	$\tilde{C}_{pm}$	$\tilde{C}_{pmk}$	Others
Kahraman and Kaya [24]	√	√			
Kaya and Kahraman [1-3, 25, 31-34]	√	√			
Kahraman and Kaya [24]	√	√			
Kahraman and Kaya [22]					√
Kaya and Kahraman [40]			√	√	√
Chen and Chen [15]					√
Hsu and Shu [27]			√		
Parchami and Mashinchi [36]	√	√	√	√	
Ramezani et al. [37]			√		
Chen et al. [38]			√		
Abdolshah et al. [39]				√	
Kaya [29]					√
Kahraman et al. [41]					√
Geramian et al. [42]	√	√			√
Parchami et al. [43]	√		√		√

4. CONCLUSIONS AND FUTURE RESEARCH SUGGESTIONS

PCIs are very useful statistics to summarize the performance of process that means how the process conforms customer expectations that can be defined as specification limits. When the literature has been analyzed, we see that the FST are widely used into PCIs. The results show that fuzzy PCIs include more information and flexibility to evaluate the process performance when it is compared with the crisp values of PCIs. As a future research suggestion, the usage of new extensions of the FST such as intiutionistic, type-2 and hesitant fuzzy can be analyzed for PCIs.

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