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MATHEMATICAL MODEL AND STATISTICAL ANALYSIS OF THE RESIDUAL STRESSES ($\sigma_{residual}$) IN THE CROSS SECTION AREA OF STEEL QUALITY PIPES J55 API 5CT

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ABSTRACT

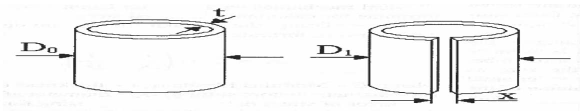
Object of this study is of the steel quality J55 API 5CT and the process of pipe forming $\varnothing 139.7\times 7.72[mm]$, $\varnothing 244.5\times 8.94[mm]$, *and* $\varnothing 323.9\times 7.10[mm]$, *with longitudinal seam pipes-ERW*
Aim of this paper is to study the impact of plastic deformation degree in the cold of residual stresses in the cross section area of steel quality pipes J55 API 5CT[1].
For the realization of this study we have used the planning method of the experiment with one-factor. We have built the mathematical model for the experiment with one index (residual stresses $\sigma_{residual}$*) and with one factor (deformation degree in the cold) and with three deformation levels. The results obtained in an experimental method are shown in the table and are processed in an analytical way while implementing the one factored experiments [2].*

Keywords: One-factor experiments, pipe, residual stresses ($\sigma_{residual}$).

1. INTRODUCTION

During the technologic production process of the longitudinal seam pipes significant factor with influence is the plastic deformation in the cold which is realized according to distorting forces on the curvature during the process of forming and calibration of pipes. It is expected that the impact to be much higher the smaller the pipe diameter is. To discover and evaluate this impact in residual stresses we made measures for three pipe diameters: Ø139.7×7.72[mm], Ø244.5×8.94[mm], and pipe Ø323.9×7.10[mm]. These three pipe profiles express three levels (1, 2 and 3) of the quality factor "deformation degree". For each level are performed four tests [3]. The slitting rings are taken from the profiles of these pipes and tests are performed while applying the criteria of the chance. The measured indicator is residual stress during forming and calibration in cross cutting of the pipes, marked with *y*. Given results (tab. 1) of the residual stresses are calculated according to the formula 1 [4].

$$\sigma_{res} = E \cdot t \left[\frac{1}{D_0} - \frac{1}{\left(\frac{x}{\pi} + D_0 \right)} \right] \qquad \ldots(1)$$



a) b)

Figure1. Schematic of the residual stress distribution in rings manufactured from tube: a) before and b) after slitting. E - modulus elasticity; t - thickness; D_0 - initial diameter; D_1 - diameter after slitting; x - net opening displacement [4].

Table 1.Results of residual stresses σ_{res} [MPa]

Reiterations/Levels	*R=162[mm]*	*R=122[mm]*	*R=70[mm]*
1	82	93.50	168.55
2	77.33	94.50	160
3	65.72	84.64	143.52
4	64.60	132.37	141
Sum y_{i+}	289.65	405	613.07 $y_{++} = 1307$
Average values $\bar{y}_{i+}$	72.41 $\bar{y}_{1+}$	101.25 $\bar{y}_{2+}$	153.26 $\bar{y}_{3+}$

2. MATHEMATICAL MODEL AND STATISTICAL ANALYSIS

2.1. Mathematical Model

Mathematical model which is predicted to reflect such a study is composed from a system by *n* equations forms [5]:

$$y_{ij} = \overline{m} + a_i + \varepsilon_{ij} \qquad \ldots (2)$$

$y_{1j} = 108.97 + (-36.56) + \varepsilon_{1j}$; $y_{2j} = 108.97 + (-7.72) + \varepsilon_{2j}$; $y_{3j} = 108.97 + 44.29 + \varepsilon_{3j}$

The formulas for calculation of round constant in which are based all observing results of index/indicator y ($\overline{m}$) and effects ($\overline{a}_i$) are:

$$\overline{m} = \frac{1}{n} \cdot y_{++} \;\; ; \;\; \overline{a}_i = \frac{1}{p} y_{i+} - \overline{m} \qquad \ldots (3)$$

Based on values from table 1 and formulas (2) we will have:

$$\overline{m} = \frac{1}{n}\cdot y_{++} = \frac{1}{12}\cdot 1307.72 = 108.97$$
$$\overline{a}_1 = \overline{y}_{1+} - \overline{y}_{++} = 72.41 - 108.97 = -36.56$$
$$\overline{a}_2 = \overline{y}_{2+} - \overline{y}_{++} = 101.25 - 108.97 = -7.72$$
$$\overline{a}_3 = \overline{y}_{3+} - \overline{y}_{++} = 153.26 - 108.97 = 44.29$$

2.2. Statistical Analysis

2.2.1. Variance Analysis

Total sum of the squares of differences (deviations) of the measured values from the average is composed by two components [2]:

$$S = S_g + S_p = 1264.25 + 13474.66 = 14738.91 \qquad \dots (4)$$

Value of summary of error squares S_g is:

$$S_g = \sum_{i=1}^{\mu}\sum_{j=1}^{p} y_{ij}^2 - \frac{1}{p}\sum_{i=1}^{p} y_{i+}^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} y_{3,4}^2 - \frac{1}{4}\sum_{i=1}^{3} y_{3+}^2 = 157093 - \frac{1}{4}623315 = 1264.25$$

In similar method we will have also the value of deviation of experimental mistake.

$$S_p = \frac{1}{p}\sum_{i=1}^{u} y_{i+}^2 - \frac{1}{\mu\times p} y_{++}^2 = \frac{1}{4}\sum_{i=1}^{3} y_{i+}^2 - \frac{1}{3\times 4} y_{++}^2 = \frac{1}{4}623315 - \frac{1}{12}1708249 = 13474.66$$

2.3. Control of Hypothesis, upon equality of the effects

For this is required control of hypothesis based on the equality of the effects a_i. According to the equation (2), hypothesis of equation of the effects H_0, will take the form [6]:

$$H_0 :\ a_1 = a_1 = \dots = a_\mu = 0 \qquad \dots (5)$$

Alternative hypothesis is:

$$H_1 :\quad a_i \neq 0 \qquad \dots (6)$$

Table 2. Summary table of variance analysis

Reason of change	*Sum of squares*	*No. of DOF*	*Average square of deviations*
Processing	$S_p = 13474.66$	$\mu - 1 = 2$	$s_p^2 = 6737.33$
Reasons of the case	$S_g = 1264.25$	$n - \mu = 9$	$s_g^2 = 140.47$
Sum of deviations	$S = 14738.91$	$n - 1 = 11$	

Value of calculated Fisher's criteria is:

$$F_{cal}=\frac{s_p^2}{s_g^2}=\frac{6737.33}{140.47}=47.96 \qquad \text{... (7)}$$

For level of importance $\alpha=0.05$ limit value of Fisher's criteria:

$$F_{tab}(\alpha);2;9=(0.05);2;9=4.26$$

$$F_{cal}=47.96>F_{tab}=4.26$$

Then, with the level of importance $\alpha=0.05$ hypothesis H_0 is rejected and effects $a_i(i=1,2,3)$ are accepted.

2.4. Comparison of the effects

2.4.1. Comparison of the effects according to minimal valid difference

To emphasize which levels are with important changes, first is required to calculate minimal valid difference $\Delta_{ik}(\alpha)$ for the level of importance $\alpha=0.05$

$$\Delta_{ik}(\alpha)=\sqrt{s_g^2\left(\frac{1}{p_i}+\frac{1}{p_k}\right)(\mu-1)\cdot F(\alpha;\mu-1,n-\mu)}=26.42$$

Based on the criteria (8) levels of effects "i" and "k" factor, so it compares a_i and a_k :

$$|\bar{a}_i-\bar{a}_k|>\Delta_{ik}(\alpha)\text{ ; } \quad |-7.72\ -\ (-36.56)|=28.84;\ 28.84>26.42$$
$$|\bar{y}_{i+}-\bar{y}_{k+}|>\Delta_{ik}(\alpha)\text{ ; } \quad |101.25-72.41|=28.84;\ 28.84>26.4 \qquad \text{… (8)}$$

From application of this criteria result that:

$|\bar{y}_{3+}-\bar{y}_{1+}|=|153.25-72.41|=80.84>26.42$ between levels 3 and 1 it has important impact

$|\bar{y}_{3+}-\bar{y}_{2+}|=|153.25-101.25|=52>26.42$ between levels 3 and 2 it has important impact

$|\bar{y}_{2+}-\bar{y}_{1+}|=|101.25-72.41|=28.84>26.42$ between levels 2 and 1 it has important impact

2.4.2. Comparison of the effects according to collective criteria of deviations

In this way "first type of mistake" to revoke a true hypothesis would be: 1- 0.857 = 0.142 (and no more 0.05). To avoid this increment of mistake we should use other criteria, Duncan's collective criteria of deviations, which will be described bellow. In case when number of experiments p in every level is same, standard mistake is calculated [2]:

$$S_{\bar{y}_{i+}}=\sqrt{\frac{1}{p}\cdot s_g^2}=\sqrt{\frac{1}{4}140.47}\ =\ 5.92 \qquad \text{… (9)}$$

By statistical tables, for $\alpha = 0.05$ and number of degrees of freedom $f = n - \mu = 12 - 3 = 9$, are with row for $q = 2,3$ valid deviation: $r_{0.05(2;9)} = 3.08$ and $r_{0.05(3;9)} = 3.23$

With valid deviations r_α and standard mistakes of levels, calculation of minimal valid deviations according to the formula:

$$R_q = r_\alpha(q, f) \cdot S_{\bar{y}_{i+}}, \qquad q = 2,3,...,\mu \qquad \text{... (10)}$$

$$r_{0.05(2;9)} = 3.08; \; r_{0.05(3;9)} = 3.23$$

$$R_2 = 3.08 \cdot 5.92 = 18.23; \; R_3 = 3.23 \cdot 5.92 = 19.12$$

Minimal valid deviation will be:

$$\bar{y}_i - \bar{y}_k \geq R_q \qquad \text{... (11)}$$

Now the comparison between levels of averages which are systematized in groups can be done:

$$\bar{y}_{3+} - \bar{y}_{1+} = 153.25 - 72.41 = 80.84 > 19.12 = R_3; \; q = 3 - 1 + 1 = 3$$

$$\bar{y}_{3+} - \bar{y}_{2+} = 153.25 - 101.25 = 52 > 18.23 = R_2; \; q = 3 - 2 + 1 = 2$$

$$\bar{y}_{2+} - \bar{y}_{1+} = 101.25 - 72.41 = 28.84 > 18.23 = R_2; \; q = 2 - 1 + 1 = 2$$

3. PROCESSING DATA WITH SOFTWARE PROGRAM DESIGN EXPERT 7

Response 1 Residual Stress
ANOVA for selected factorial model
Analysis of variance table [Classical sum squares – Type II]

Source	Sum of Squares	df	Mean Square	F	p – value Value Prob > F
Model	13328.67	2	6664.33	29.16	0.0001 significant
A- Defor. Degree	13328.67	2	6664.33	29.16	0.0001
Pure Error	2057.00	9	228.56		
Cor Total	15385.67	11			

The Model F-value of 29.16 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A are significant model terms.

Std. Dev.	15.12	R-Squared	0.8663
Mean	108.83	Adj R-Squared	0.8366
C.V. %	13.89	Pred R-Squared	0.7623
PRESS	3656.89	Adeq Precision	10.650

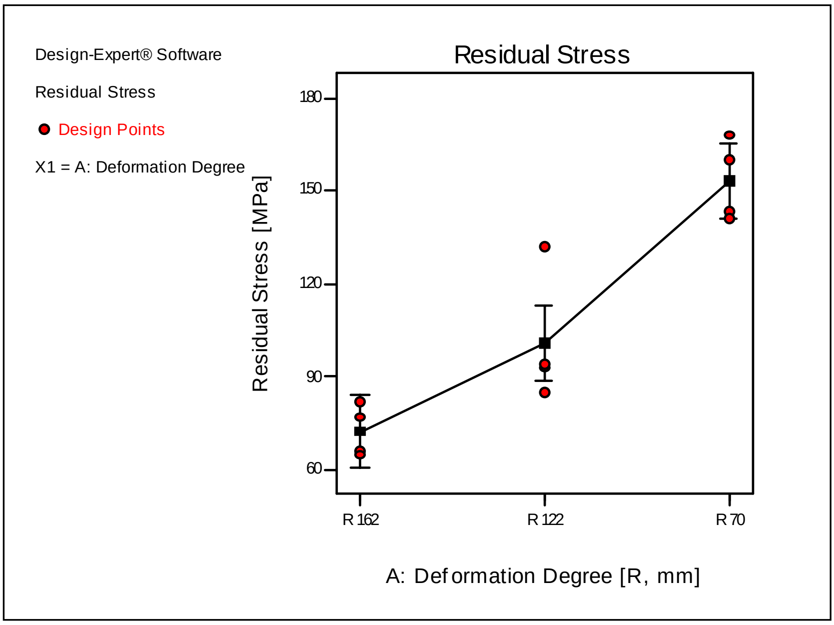
The "Pred R-Squared" of 0.7623 is in reasonable agreement with the "Adj R-Squared" of 0.8366.
"Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your

ratio of 10.650 indicates an adequate signal. This model can be used to navigate the design space.

Treatment Means

Treatment	Mean Difference	df	Standard Error	t for H_0 Coeff=0	Prob > |t|
1 vs 2	-28.50	1	10.69	-2.67	0.0258
1 vs 3	-80.50	1	10.69	-7.53	<0.0001
2 vs 3	-52.00	1	10.69	-4.86	0.0009

Values of "Prob > |t|" less than 0.0500 indicate the difference in the two treatment means is significant.



4. CONCLUSION

In three applied methods (criteria) for results analysis, with degree of decreasing the mistake of the first type, from 0.142, in 0.05 and in p = 0.0001, are confirming the forming of pipes, the deformation degree throughout the bending of sheet and calibration in the cold influences in the increase of residual stresses. The influence of the impact is much higher the smaller the pipe diameter is.

5. REFERENCES

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