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FUZZY ESTIMATIONS OF PROCESS INCAPABILITY INDEX

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SUMMARY
Process capability indices (PCIs) provide numerical measures on whether a process conforms to the defined manufacturing capability prerequisite. In the literature some PCIs have been used to measure the ability of process to decide how well the process meets the specification limits (SLs). These have been successfully applied by companies to compete with and to lead high-profit markets by evaluating the quality and productivity performance. In this paper, one of the most important PCIs, process incapability index $\left(C_{pp}\right)$ *which is easy to apply, and provides more process information than other process PCIs is analyzed together with the indices inaccuracy* $\left(C_{ia}\right)$ *and imprecision* $\left(C_{ip}\right)$ *In this paper, the index* C_{pp} *is analyzed by using the fuzzy set theory to obtain a deep and flexible analysis. To produce fuzzy process incapability index* $\left(\tilde{C}_{pp}\right)$*, fuzzy process mean* $(\tilde{\mu})$ *and fuzzy variance* $\left(\tilde{\sigma}^2\right)$*, which are obtained by using the fuzzy extension principle, are used together with fuzzy SLs (SL□s) and fuzzy target value (T□). In order to find the membership functions of fuzzy inaccuracy index* $\left(\tilde{C}_{ia}\right)$ *and fuzzy imprecision index* $\left(\tilde{C}_{ip}\right)$*, the α-cuts of the fuzzy observation are employed. Then the fuzzy estimations of the index* $\tilde{C}_{pp}$ *are produced for triangular fuzzy numbers (TFN). The proposed index* $\tilde{C}_{pp}$ *is applied in a piston manufacturer firm.*

Keywords: Process incapability index, inaccuracy index, imprecision index, fuzzy set theory, decision making

1. INTRODUCTION

Process capability indices (PCIs) are used widely in many different industries to determine whether or not a manufacturing process can produce articles within the specified limits. A process meeting customer requirements is called "capable". A process capability index (PCI) is a process characteristic relative to specifications. These indices help us to decide how well a process meets specifications. The most commonly used PCIs are C_p, C_{pl}, C_{pu}, C_{pk}, and C_{pm}. If the process characteristic X is independently and identically distributed by a normal distribution with a mean, μ, and a standard deviation, σ, these indices are defined as follows (Kotz and Johnson, 2002; Montgomery, 2005; Kaya and Kahraman, 2010):

$$C_p=\frac{USL-LSL}{6\sigma},\ C_{pl}=\frac{\mu-LSL}{3\sigma},\quad C_{pu}=\frac{USL-\mu}{3\sigma},\quad C_{pk}=\min\left\{C_{pl},C_{pk}\right\} \tag{1}$$

where USL and LSL are the upper and lower specification limits, respectively.
The C_p index measures only the potential of a process to produce acceptable products, and does not take into account where the process is centered. C_p can not reflect the tendency of

process centering and thus gives no indication of the actual process performance. Generally, the C_{pk} index takes into account the process variation as well as the location of the process mean relative to the specification limits. The C_{pk} index is computed using both location and dispersion information about the process. The index C_{pk} is the shorter standardized distance from the center of the process distribution to either LSL or USL. C_p and C_{pk} indices do not take into account the cost of failing to meet customer's requirements. The C_{pm} index can be used for this reason. Chan et al. (1988) developed the C_{pm} index, which provides the indicators of both process variability and deviation of process mean from target value, and also provides a quadratic loss interpretation, taking into account the process departure.

PCIs are summary statistics which measure the actual or the potential performance of the process characteristics relative to the target and specification limits. Many alternative capability indices were described in the literature for several reasons (Kotz and Johnson, 2002; Montgomery, 2005; Kaya and Kahraman, 2010).

The index C_{pp} provides additional and individual information concerning the process accuracy and the process precision. It has been widely used to provide numerical measures on whether a production is capable of producing items within the specification limits preset by the designer (Chen and Chen, 2008). In this paper, this index is analyzed under fuzziness. Zadeh (1965) introduced fuzzy set theory that is a branch of mathematics that allows a computer to model the real world in the same way that people do to overcome some uncertain and vague concepts in variables. The fuzzy set theory has a wide application in many fields since it provides a simple way to reason with vague, ambiguous, and imprecise input or knowledge. The fuzzy set theory has also been applied in process capability analysis to provide more information and more sensitiveness on PCIs. The fuzzy set theory brings an advantage to easily define main characteristics of PCIs by using linguistic variables. Although PCIs have been analyzed under fuzziness, the literature has no research about fuzzy estimation of process incapability index $\left(\tilde{C}_{pp}\right)$. So this paper investigates the index $\tilde{C}_{pp}$ by obtaining fuzzy estimations of inaccuracy index $\left(\tilde{C}_{ia}\right)$ and imprecision index $\left(\tilde{C}_{ip}\right)$. The fuzzy estimations of the index $\tilde{C}_{pp}$ are produced for triangular fuzzy numbers (TFN) by using fuzzy specification limits $\left(S\tilde{L}s\right)$, fuzzy mean $(\tilde{\mu})$, fuzzy variance $\left(\tilde{\sigma}^2\right)$ and fuzzy target value $\left(\tilde{T}\right)$.

The rest of this paper is organized as follows: The process incapability index is briefly introduced in Section 2. The fuzzy estimations of process incapability index $\left(\tilde{C}_{pp}\right)$ are produced in Section 3. Section 4 includes a real case application for a piston manufacturer firm. The conclusions and future research directions are discussed in Section 5.

2. PROCESS INCAPABILITY INDEX

The indices C_p and C_{pk} are most used PCIs to measure process ability. Generally, the C_{pk} index takes into account the process variation as well as the location of the process mean relative to the specification limits while the C_p index reflects only the magnitude of the process variation. Kane (1986) investigated comprehensively these indices and their estimators. Greenwich and Jahr-Schaffrath (1995) defined the index C_{pp} which provides an uncontaminated separation between information concerning process accuracy and process precision as follow:

$$C_{pp} = C_{ia} + C_{ip} \tag{2}$$

where the inaccuracy index $C_{ia} = \left(\frac{\mu - T}{D}\right)^2$ and imprecision index $C_{ip} = \left(\frac{\sigma}{D}\right)^2$. Thus,

$$C_{pp}=\left(\frac{\mu-T}{D}\right)^2+\left(\frac{\sigma}{D}\right)^2 \quad (3)$$

where $D=\frac{\min\{T-LSL,USL-T\}}{3}=\frac{d^*}{3}$ (4)

The index C_{pp} can be widely used to summarize process performance since it simultaneously detects process inaccuracy and process imprecision by using both of the indices C_{ia} and C_{ip}. The index C_{pp} contains the information namely, inaccuracy (the departure of the process mean, μ, from the target value, T) and imprecision (the magnitude of the process variation, σ^2). Moreover, the index C_{pp} provides an uncontaminated separation between information concerning the process accuracy and precision. The index C_{pp} assumes a smaller value for a process more capable of meeting its specifications and a larger value for a less capable process. A process is most capable when $C_{pp}=0$. For this, the process mean must be at the process target (μ = T) and the process variance must be zero (σ^2 = 0). Any non-zero value of C_{pp} indicates some degree of incapability of the process. Thus, C_{pp} is a process incapability index (Greenwich and Jahr-Schaffrath, 1995). In this paper, the fuzzy estimations of index $\tilde{C}_{pp}$ are produced by obtaining fuzzy estimations of the indices $\tilde{C}_{ia}$ and $\tilde{C}_{ip}$.

3. FUZZY PROCESS INCAPABILITY INDEX

The fuzzy logic is a matter of the fuzzy set theory particularly used to dealing with imprecise information by using membership function and was formalized by Zadeh (1965). In a classical set, an element belongs to, or does not belong to, a set whereas an element of a fuzzy set naturally belongs to the set with a membership value from the interval [0, 1]. Fuzzy set theory has been studied extensively over the past 30 years since it gives an advantage to define the parameters more flexible and to analyze the results with more sensitiveness. Although most of the early interest in fuzzy set theory was representing of uncertainty in human cognitive processes, it is now applied to problems in engineering, business, health sciences, economic, and the natural sciences. Over the last years there have been successful applications of the fuzzy set theory in quality management and assurance. After the inception of fuzzy sets in statistical process control, some studies have been made to combine PCIs and the fuzzy set theory. In this paper, fuzzy estimations of the index $\tilde{C}_{pp}$, $\tilde{C}_{ia}$ and $\tilde{C}_{ip}$ are derived for the first time by using $\tilde{\mu}$, $\tilde{\sigma}^2$ together with $S\tilde{L}s$ and $\tilde{T}$ values. In this paper, the Buckley's fuzzy estimation method (Buckley and Eslami, 2004; Buckley 2004) is used to obtain the membership functions of the $\tilde{\mu}$ and $\tilde{\sigma}^2$ as detailed below.

3.1. Membership Function of Fuzzy Process Variance and Fuzzy Process Mean

Let x be a random variable which has a probability density function, $N(\mu,\sigma^2)$, with unknown mean (μ) and unknown variance (σ^2). A random sample $x_1,x_2,...,x_n$ from $N(\mu,\sigma^2)$ can be taken to estimate σ^2. The mean of this sample is a crisp number $(\bar{x})$. Also it is known that $\frac{(n-1)s^2}{\sigma^2}$ has a "*chi-square* distribution" with *n-1* degrees of freedom. The triangular fuzzy membership functions of variance can be obtained as follows (Buckley and Eslami, 2004; Buckley 2004):

$$\tilde{\sigma}^2(\alpha)=\left[\frac{(n-1)s^2}{[1-\alpha]\chi^2_{R,0.005}+(n-1)\alpha},\frac{(n-1)s^2}{[1-\alpha]\chi^2_{L,0.005}+(n-1)\alpha}\right],\ 0.01\le\alpha\le1 \quad (5)$$

Then the fuzzy estimation of μ can be obtained as follows:

$$\mu_{l,\tilde{\sigma}}(\alpha)=\min\left(\bar{x}-z_{\alpha/2}\frac{\sigma_i(\alpha)}{\sqrt{n}}\right) \qquad i=1,2 \tag{6}$$

$$\mu_{r,\tilde{\sigma}}(\alpha)=\max\left(\bar{x}+z_{\alpha/2}\frac{\sigma_i(\alpha)}{\sqrt{n}}\right) \qquad i=1,2 \tag{7}$$

where $\sigma_1(\alpha)$ and $\sigma_2(\alpha)$ represent the left and right sides of $\tilde{\sigma}^2$, respectively and can be calculated by using Eq. (5).

3.2. Fuzzy estimation of the process incapability index $(\tilde{C}_{pp})$

In this paper the fuzzy estimations of index $\tilde{C}_{pp}$ is obtained by using the fuzzy estimations of index $\tilde{C}_{ia}$ and index $\tilde{C}_{ip}$. The α-cuts for $\tilde{C}_{pp}$ can be written based on α-cuts of the indices $\tilde{C}_{ia}$ and $\tilde{C}_{ip}$ as in Eq. (8):

$$\tilde{C}_{pp}(\alpha)=\tilde{C}_{ia}(\alpha)+\tilde{C}_{ip}(\alpha)=\left\{\tilde{C}^L_{ia}(\alpha)+\tilde{C}^L_{ip}(\alpha),\ \tilde{C}^R_{ia}(\alpha)+\tilde{C}^R_{ip}(\alpha)\right\} \tag{8}$$

The α-cuts for the indices $\tilde{C}_{ia}$ and $\tilde{C}_{ip}$ can be obtained by using Eqs. (9) and (10).

$$\tilde{C}_{ia}(\alpha)=\left\{C^L_{ia}(\alpha),C^R_{ia}(\alpha)\right\}=\left\{\left(\frac{\mu_{l,\tilde{\sigma}}(\alpha)-\tilde{T}_r(\alpha)}{\tilde{D}_r(\alpha)}\right)^2,\left(\frac{\mu_{r,\tilde{\sigma}}(\alpha)-\tilde{T}_l(\alpha)}{\tilde{D}_l(\alpha)}\right)^2\right\} \tag{9}$$

$$\tilde{C}_{ip}(\alpha)=\left\{C^L_{ip}(\alpha),C^R_{ip}(\alpha)\right\}=\left\{\frac{\tilde{\sigma}^2_l(\alpha)}{\tilde{D}^2_r(\alpha)},\frac{\tilde{\sigma}^2_r(\alpha)}{\tilde{D}^2_l(\alpha)}\right\} \tag{10}$$

Suppose we have a fuzzy process for which the target value, upper and lower specification limits are defined as $\tilde{T}=\text{TFN}(t_1,t_2,t_3)$, $U\tilde{S}L=\text{TFN}(u_1,u_2,u_3)$, and $L\tilde{S}L=\text{TFN}(l_1,l_2,l_3)$. The left and right sides of the membership function for $\tilde{C}_{ia}$ can be obtained TFN as follows:

$$U\tilde{S}L(\alpha)=\left[(u_2-u_1)\alpha+u_1,(u_2-u_3)\alpha+u_3\right],L\tilde{S}L(\alpha)=\left[(l_2-l_1)\alpha+l_1,(l_2-l_3)\alpha+l_3\right]$$
$$\tilde{T}(\alpha)=\left[(t_2-t_1)\alpha+t_1,(t_2-t_3)\alpha+t_3\right] \tag{11}$$

$$\tilde{C}^L_{ia}(\alpha)=\left(\frac{\min\left(\bar{x}-z_{\alpha/2}\frac{\sigma_i(\alpha)}{\sqrt{n}}\right)-(t_2-t_3)\alpha-t_3}{\frac{\min\left\{(u_2-u_3-t_2+t_1)\alpha+u_3-t_1,(t_2-t_3-l_2+l_1)\alpha+t_3-l_1\right\}}{3}}\right)^2 \tag{12}$$

$$\tilde{C}^R_{ia}(\alpha)=\left(\frac{\max\left(\bar{x}+z_{\alpha/2}\frac{\sigma_i(\alpha)}{\sqrt{n}}\right)-(t_2-t_1)\alpha-t_1}{\frac{\min\left\{(u_2-u_1-t_2+t_3)\alpha+u_1-t_3,(t_2-t_1-l_2+l_3)\alpha+t_1-l_3\right\}}{3}}\right)^2 \tag{13}$$

The left and right sides of the membership function for $\tilde{C}_{ip}$ can also be obtained by using Eqs. (14) and (15):

$$\tilde{C}^L_{ip}(\alpha)=\left(\frac{\frac{(n-1)s^2}{(1-\alpha)\chi^2_{R,0.005}+(n-1)\alpha}}{\left(\frac{\min\left\{(u_2-u_3-t_2+t_1)\alpha+u_3-t_1,(t_2-t_3-l_2+l_1)\alpha+t_3-l_1\right\}}{3}\right)^2}\right) \tag{14}$$

$$\tilde{C}_{ip}^{R}(\alpha)=\left(\frac{\dfrac{(n-1)s^2}{(1-\alpha)\chi^2_{L,0.005}+(n-1)\alpha}}{\left(\dfrac{\min\left\{(u_2-u_1-t_2+t_3)\alpha+u_1-t_3,(t_2-t_1-l_2+l_3)\alpha+t_1-l_3\right\}}{3}\right)^2}\right) \quad (15)$$

As seen from above equations, it is a necessity to compare fuzzy values for determining the minimum value. It is possible to meet many ranking methods in the literature. In this paper a defuzzication method named the total integral value method with an index of optimism $\omega\in[0,1]$ and proposed by Liou and Wang (1992) is used.

4. APPLICATION

In this paper, fuzzy process incapability index with asymmetric tolerances is applied in a piston manufacturer firm from Konya's Industrial Area, Turkey. The firm produces piston, liner and piston ring (Kaya, 2009a; 2009b). Process engineers of the firm decide to apply the index $\tilde{C}''_{pp}$ to increase not only the flexibility of process but also the sensitiveness of the results for some characteristics of a Volvo Marine motor's piston for diesel engine. The measurable characteristics of the piston such as piston diameter, compression height, bowl depth, total length, combustion chamber diameter, pin diameter, and pin length are monitored. The index $\tilde{C}''_{pp}$ has been applied for the characteristic "compression height". For this aim, the process engineers take 300 samples and determine the mean and variance of compression height as $\bar{x}=114.206$, $s^2=0.0002$. The fuzzy estimations of process mean and variance are calculated as $\mu_{\tilde{\sigma}}=(114.204,\ 114.206,\ 114.208)$, $\tilde{\sigma}^2=(0.000164,\ 0.0002,\ 0.00025)$, respectively. It is clear that the fuzzy estimations of process mean and variance not only include more information but also the crisp values $\bar{x}=114.206$ and $s^2=0.0002$ with a membership value of 1.00, respectively.
In process capability analysis, the specification limits are determined by using fuzzy terms. The upper and lower specification limits for compression height are defined as "*Approximately 114.220*" and "*Approximately 114.190*", respectively. The target value for compression height is also defined as "*Approximately 114.203*". These definitions are converted to TFNs by using Eqs. (11) to calculate the index $\tilde{C}''_{pp}$.

The fuzzy estimations of process inaccuracy and imprecision indices are calculated as $\tilde{C}_{ia}=(0.0012,\ 0.4793,\ 2.1286)$, $\tilde{C}_{ip}=(7.5323,\ 10.6509,\ 15.5236)$, respectively. The index $\tilde{C}_{pp}$ is determined as $(7.534,\ 11.13,\ 17.652)$ and the membership function of the index $\tilde{C}''_{pp}$ is illustrated in Figure 1.

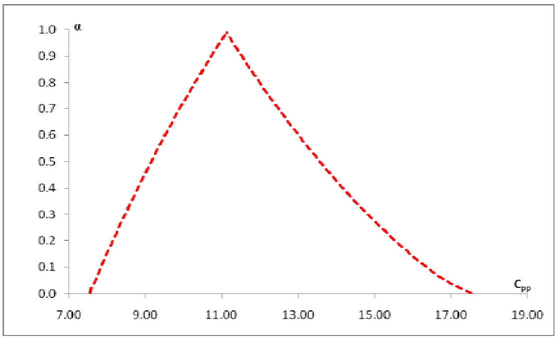

Figure 1. The membership function of the index $\tilde{C}_{pp}$

We know that if the value of process incapability index is bigger than 1.00, the process can be classified as "Inadequate". We can easily interpret that the piston production process cannot be classified "capable" for the characteristic, "compression height". As it seen from Figure 1, the fuzzy analysis of process incapability index takes into account all possible values together with related degree of membership. This gives an advantage by obtaining more detailed and flexible analysis both to increase process quality and to decrease process variation. We can obtain a result that the process has no possibility anyway to be classified as "capable" with respect its fuzzy values.

5. CONCLUSION

The process capability analysis can be defined as a measure of inherent variability in a process as compared to the requirements of the product which defined by using specification limits.
There have been proposed numerous process capability indices in the literature and they have been popularly used in industry to provide a unitless measure on whether a process is capable. In this paper, one of the most important process capability indices, process incapability index $\left(C_{pp}\right)$ which is easy to apply, and provides more process information than other process PCIs is analyzed together with indices inaccuracy $\left(C_{ia}\right)$ and imprecision $\left(C_{ip}\right)$ for a piston production process.

To improve the sensitiveness and flexibility of process incapability index, the fuzzy set theory is integrated. For this aim, the fuzzy estimations of process mean $(\tilde{\mu})$ and variance $\left(\tilde{\sigma}^2\right)$ are produced and then fuzzy estimations of inaccuracy index $\left(\tilde{C}_{ia}\right)$ and imprecision index $\left(\tilde{C}_{ip}\right)$ are obtained. Then the fuzzy estimations of the index $\tilde{C}_{pp}$ are derived

The proposed fuzzy incapability index is used to evaluate the quality conditions of piston production process with respect to a characteristic compression height. The obtained fuzzy results show that fuzzy indices include more information about quality condition. The results also show that the process quality is "Inadequate" for all possible values of process incapability index.
For the future research, the fuzzy estimation of process incapability index can also be obtained by taking into account other type of fuzzy numbers such as trapezoidal etc.

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