VIBRATION MEASUREMENTS FOR COMPOSITE PLATE: FINITE ELEMENT METHOD AND ESPI

Soňa Rusnáková, Martina Mokryšová, Juraj Slabeycius, Dana Bakošová, Ivan Letko
Faculty of Industrial Technologies in Púchov
Departement of Pysical Engeeniering of Materials
University of Alexander Dubček in Trenčín
I.Krasku 491/30, Púchov
Slovak Republic, www.fpt.tnuni.sk
e-mail: rusnakova@fpt.tnuni.sk

SUMMARY
Composite materials have a large number of parameters associated with their manufacturing. It is not physically possible to control all these parameters, and hence variation in the material properties results. A vibration finite element model of laminated composite material plates has been developed, using the finite element commercial software Cosmos. The validation has been made with prepreg M 34 square plates. For this first case the numerical simulations have been compared with experimental solutions by using electronic speckle pattern interferometry (ESPI).

Keywords: Electronic speckle pattern interferometry, composite materials, FEM.

1. INTRODUCTION

Fibre-reinforced components of various shapes and different boundary conditions (free, clamped, and hinged) commonly occur in practice. Designers need to be able to predict the stiffness parameters and damping values of such components for conditions such as aeroelasticity, acoustic fatigue, and so on. Electronic speckle pattern interferometry (ESPI) can be useful tool determination resonant frequencies. Subsequently for example it is possible to make various computed models and compare values obtained from ESPI measurements with FEM analyses.

Plates are structural elements of great importance and are used extensively in all fields of engineering applications such as aerospace and electronic industry. There have been extensive studies on the vibration of classical plates for various shapes, boundaries, and loading conditions for nearly two centuries. The analysis methods for vibrations of plates can be classifed into three types, which are analytical [1], numerical [2,3] and experimental [4].

2. OUT-OF-PLANE VIBRATION

When the specimen vibrates periodically, the first image is recorded as a reference. The light intensity of this reference as image detected by a charge-coupled device (CCD) camera can be expessed by the time-averaged method as:

$$I_1 = \frac{1}{\tau}\int_0^{\tau}\left\{ I_A + I_B + 2\sqrt{I_A I_B} \times \cos\left[\phi + \frac{2\pi}{\lambda}\left(1+\cos\theta\right)A\cos\omega t\right]\right\}dt \tag{1}$$

Where I_A= the object light intensity, I_B= the reference light intensity, τ=object light intensity, ϕ=the phase difference between object and reference light, λ=the wavelength of laser, θ=the angle between object light and observation direction, A=the vibration amplitude, and ω=the angular frequency.

Let $\Gamma = \left(2\pi/\lambda\right)\left(1+\cos\theta\right)$ and $\tau = 2m\pi/\omega$, where *m* is an integer. Then Eq. (1) can be written as:

$$I_1 = I_A + I_B + \frac{2\sqrt{I_A I_B}}{\tau}\int_0^{\tau}\cos\left(\phi + \Gamma A\cos\omega t\right)dt =$$

$$I_A + I_B + \frac{2\sqrt{I_A I_B}}{\tau}\mathrm{Re}\left\{e^{i\phi}\int_0^{\tau}\exp\left(i\Gamma A\cos\omega t\right)dt\right\}$$

where *Re* stands for the real part. Since $e^{iz\sin\alpha} = \sum_{-\infty}^{\infty} J_n\left(z\right)e^{in\alpha}$ (*Jn* is a Bessel function of the first kind of order *n*), hence we have

$$\mathrm{Re}\left\{e^{i\phi}\int_0^{\tau}\exp\left(i\Gamma A\cos\omega t\right)dt\right\} = \mathrm{Re}\left\{e^{i\phi}\sum_{-\infty}^{\infty}J_n\left(\Gamma A\right)\int_0^{\tau}e^{in(\pi/2-\omega t)}dt\right\}$$

$$= \mathrm{Re}\left\{e^{i\phi}\sum_{-\infty}^{\infty}J_n\left(\Gamma A\right)e^{in(\pi/2-\omega t)}\frac{\left(e^{-in\omega\tau}-1\right)}{-in\omega}\right\}$$

However all the terms will be zero except *n=0* . Finally Eq. (1) can be expressed as

$$I_1 = I_A + I_B + 2\sqrt{I_A I_B}\left(\cos\phi\right)J_0\left(\Gamma A\right) \tag{2}$$

where J_0 is a zero order Bessel function of the first kind. After image processing and rectifying, the intensity of the first can be expressed as

$$I_1 = I_A + I_B + 2\sqrt{I_A I_B}\left|\left(\cos\phi\right)J_0\left(\Gamma A\right)\right| \tag{3}$$

As the vibration of the specimen continues, we assume that the vibration amplitude chances from A to $A+\Delta A$ because of he electronic noise or instability of the apparatus. The light intensity of the second image can be represented as

$$I_2 = \frac{1}{\tau}\int_0^{\tau} I_A + I_B + \left\{2\sqrt{I_A I_B}\cos\left[\phi + \Gamma\left(A+\Delta A\right)\cos\omega t\right]\right\}dt \tag{4}$$

When the vibration amplitude variation ΔA is rather small, Eq. (4) can be expanded in Taylor series. By keeping the first two terms and neglecting the higher-order terms, we rewrite Eq. (4) as follows:

$$I_2 = I_A + I_B + 2\sqrt{I_A I_B}\left(\cos\phi\right)\left[1-\tfrac{1}{4}\Gamma^2(\Delta A)^2\right]J_0\left(\Gamma A\right) \tag{5}$$

By image processing and rectifying, I_2 can be similarly expressed as

$$I_2 = I_A + I_B + 2\sqrt{I_A I_B}\left|\left(\cos\phi\right)\left[1-\tfrac{1}{4}\Gamma^2(\Delta A)^2\right]J_0\left(\Gamma A\right)\right| \tag{6}$$

When these two images (the first and second images) are subtracted by the image processing system, i.e., Eq. (3) is subtracted from Eq. (6), and is rectified; the resulting image intensity can be expressed as

$$I = I_2 - I_1 = \frac{\sqrt{I_A I_B}}{2}\left|\left(\cos\phi\right)\Gamma^2\left(\Delta A^2\right)J_0\left(\Gamma A\right)\right|. \tag{7}$$

3. METHOD

The schematic layout of ESPI optical system, as shown in Fig.1, is employ to perform the out-of-plane vibration measurement of the resonant frequencies and mode shape for composite plate.

Composite plates with all edges free are used to have the ideal boundary conditions for experimental simulation. The resonant frequency and correspondent mode shape for the vibrating plate are determined experimentally using the no contacting optical method ESPI. A He-Ne laser with wavelength λ = 632,8 nm is used as the coherent light source. The laser beam is divided into two parts, the reference and object beam, by a beamsplitter. The object beam travels to the specimen and then reflects to the CCD camera via the mirror and reference plate. The CCD camera converts the intensity distribution of the interference pattern of the object into a corresponding video signal. The signal is electronically processed and finally converted into an image on the video monitor.

The experimental procedure of ESPI technique is performed as follows. First, a reference image is taken, after the specimen vibrates, then the second image is taken, and the reference image is subtracted by the image processing system.

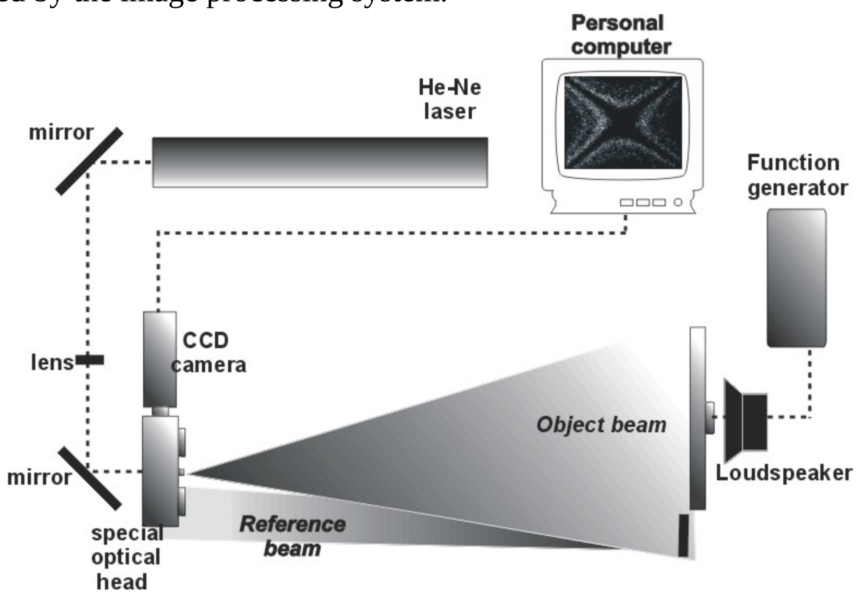


Fig.1 Schematic diagram of ESPI setup for out-of-plane measurements.

Then the function generator is carefully and slowly turned, the number of fringes will increase and the fringe pattern will become clearer as the resonant frequency is approached. The resonant frequencies and corresponding mode shapes can be determined at the same time using the ESPI optical system.

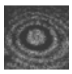
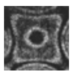
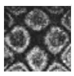
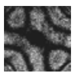
4. EXPERIMENTAL RESULTS

In many cases, experimentaland computational methods can be combined so that the data obtained by one method can be used by the other one to verify the results [5]. In this paper we are interested in comparing the results obtained by means of a computational method – FEM with those obtained experimentally using electronic speckle pattern interferometry. As a test object we used a square plate excited to different resonant vibrations by a sinusoidal acoustic source.

Table 1. Description of investigated sample with different thickness.

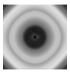
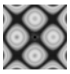
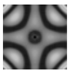
Investigated Samples	1	2	3	4
Thickness *h* [mm]	0,8	1,05	1,35	1,65
Number of layers	3	4	5	6

Table 2. Resonant frequencies obtanined from ESPI.

			Thicknes *h* [mm]			
	Mode		**0,8**	**1,05**	**1,35**	**1,65**
Resonant frequeny [Hz]	1		61	78	93	112
	2		138	187	228	293
	3		179	248	309	378
	4		374	522	663	800
	5		432	620	758	919
	6		498	694	861	1058
	7			795	983	1200

The investigated plate was made from pre-preg, *M 34* (epoxy/glass). Pre-preg is a term for "pre-impregnated" composite fibres. These usually take the form of a weave or are uni-directional. They already contain an amount of the matrix material used to bond them together and to other components during manufacture. The dimension of investigated plate was 175x175 mm with various thickness like it can be seen in the Table 1. Material parameter of pre-preg is $\rho = 1665\ kg/m^3$, Young's modulus $E = 21\ GPa$ a Poisson ratio $\mu=0,13$. Boundary conditions are in the centre fixed by special stand and the edges are free.

Table 3. Resonant frequencies obtanined from FEM (COSMOS).

		Thicknes *h* [mm]			
	Mode	**0,8**	**1,05**	**1,35**	**1,65**
Resonant frequeny [Hz]	1 	48	63	81	99
	2 	167	219	282	345
	3 	210	276	355	434
	4 	403	530	681	832
	5 	512	672	864	1055
	6 	515	676	869	1062
	7 	626	821	1056	1290

5. CONCLUSION

In this paper investigated changes of resonant frequencies obtained by ESPI with various thickness of investigated prepreg samples.
With increasing of thickness of plate is increasing resonant frequencies of corresponding mode shapes, too. It can be seen in Table 2. Experimentally obtained values resonant frequencies are corresponding with values obtained by FEM. In these two cases increasing resonant frequencies nearly linearly what is confirmed the principle: with increasing of stiffness of sample we need higher loaded frequency. The results presented in Tables 2 and 3 shows generally good agreement between the numerically predicted and experimentally measured resonant frequencies.

ACKNOWLEDGMENTS
The authors gratefully acknowledges the financial support of this research by the Grant Ministry of Education Slovak Republic AV /2008.

6. REFERENCES

[1] Harik, I.E., Liu, X., Balakrishnan, N.: Analytic solution to free vibration of rectangular plates, Journal of Sound and Vibration 153, 51–62, 1992.

[2] Chen, W.Q., Xu,R.Q., Ding, H.J.: On free vibration of a piezoelectric composite rectangular plate, Journal of Sound and Vibration 218, 741–748, 1998.

[3] Sakiyama, T., Matsuda. H.: Free vibration of rectangular Mindlin plate withmixed boundary conditions, Journal of Sound and Vibration 113, 208–214, 1987.

[4] Low, K.H., Chai, G.B., Lim, T.M., Sue, S.C.: Comparisons of experimental and theoretical frequencies for rectangular plates with various boundary conditions and added masses, International Journal of Mechanical Sciences 40, 1119–113, 1998.

[5] Caldersmith, G.W.: Acoustica, 56, 144., 1984.