A CONTRIBUTION TO THE STUDY OF TRANSIENT HEAT TRANSFER ANALYSIS

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SUMMARY

In this study, a macroscopic transient heat transfer analysis of the cooling process of the solid, based on the generalized lumped thermal capacitance approach is described. An analyzed macroscopic combined heat transfer by convection and radiation in the cooling process of the solid is modeled by using a generalized thermal lumped capacitance approximation with dynamic effective heat transfer coefficient for combined convective and radiative cooling. An exact analytical solution of analyzed combined transient heat transfer problem is done.

Keywords: transient heat transfer, lumped capacitance model

1. INTRODUCTION

The lumped capacitance approach is a well known method of macroscopic transient processes analysis in the science and egineering because of its innate simplicity [1, 2]. In the area of a transient heat transfer processes analysis it assumes that the temperature in the material is spatially uniform at any instant of time during a heat transfer process, i.e. the temperature of material can be taken to be a function of time only. Heat transfer analysis which utilizes this theoretical idealization is applicable only when the *Biot number* (the ratio of internal thermal resistance within the material to external thermal resistance at its surface) is less than or equal to *0,1*. The lumped thermal capacitance analysis is based on the solution of a first order differential total energy balance equation for the material which must relate the rate of heat loss at its surface, to the rate of change of the internal energy of material. The *simplest* lumped thermal capacitance model for the material regards only the convective heat transfer between the solid and its surroundings. For the homogeneous solid with constant physical, spatial and thermal properties, that is being cooled from the initial temperature T_0 by a fluid with constant temperature T_∞ at the constant convective heat transfer coefficient h_c, the solution of the total energy balance differential equation has a form of an exponential temperature function

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left(-\frac{h_c S t}{\rho c_p V}\right), \tag{1}$$

which predicts a time history of the solid temperature during a cooling process. Quantities ρ, c_p, V and S are density and specific heat of the solid material, volume and surface of the solid, respectivelly and t is a time variable. The quantity $\rho c_p V/h_c S$ is called *the thermal time constant* for the geometry and has dimensions of time. The numerator of the time constant is called *lumped thermal capacitance* of the solid, and the ratio $1/h_c S$ is recognized as the *convective resistance* [3].
However, a more complex approach to the transient heat transfer problems solution offers a *generalised* lumped thermal capacitance model which allows for a *combined* heat tranfer by convection and radiation [4] what more accurately describes the real transient heat transfer phenomenon. The aim of the study is a more detailed analysis of such model for the material cooled by the fluid, because of just a cooling processes prediction plays very important role in many areas of science, research, material engineering, as well as in the industrial practice.

2. GENERALIZED LUMPED CAPACITANCE MODEL

The total energy balance equation for the solid cooled by combined heat transfer through convection with constant convective heat transfer coefficient h_c and by radiation

$$\rho c_p V \frac{dT(t)}{dt} = -h_c S\left[T(t) - T_\infty\right] - \varepsilon\sigma S\left[T(t)^4 - T_\infty^4\right] \tag{2}$$

represents an ordinary first order nonlinear nonhomogeneous differential equation without exact analytical solution for $T(t)$, where ε is an *emissivity coefficient* of the solid surface and $\sigma = 5{,}627.10^{-8}\ W.m^{-2}.\ K^{-4}$ is *Stefan-Boltzmann constant* [5]. However, since

$$T(t)^4 - T_\infty^4 = \left[T(t) - T_\infty\right]\left[T(t) + T_\infty\right]\left[T(t)^2 + T_\infty^2\right] \tag{3}$$

one can see, that if temperature difference $T(t) - T_\infty$ is resonably small compared to T_∞, then

$$T(t)^4 - T_\infty^4 \approx 4T_\infty^3\left[T(t) - T_\infty\right]. \tag{4}$$

Consequently, energy balance equation (2) can be rewritten to the form of

$$\rho c_p L \frac{dT(t)}{dt} \approx -h\left[T(t) - T_\infty\right], \tag{5}$$

where

$$h \approx h_c + 4\varepsilon\sigma T_\infty^3 \tag{6}$$

is an constant *effective coefficient* for combined convective and radiative cooling and

$$L = \frac{V}{S} \tag{7}$$

is a *characteristic size* of the solid. Following the relation (6) it can be concluded, that the cooling by radiation plays a major role only at such coolig conditions when the heat transfer coefficient corresponding to the pure convective cooling is not significantly greater than the value of quantity $4\varepsilon\sigma T_\infty^3$, i.e. at combined heat transfer by natural convection and radiation at temperatures greater then $0\ °C$. An exact analytical solution of the linear nonhomogeneous differential equation (5) has the analogical form as the exponential temperature funcion of simplest thermal lumped capacitance model (1) and it is aplicable at the same condition of the sufficiently small Biot number.

3. DYNAMIC HEAT TRANSFER COEFFICIENT

At greater temperature differences between solid and its surrounding the aproximation (6) can not be used. The effective coefficient for convective and radiative heat transfer h receives a form of temperature-dependent function

$$h(T) = h_c + \varepsilon\sigma \left[T(t) + T_\infty\right]\left[T(t)^2 + T_\infty^2\right], \tag{8}$$

energy balance equation (5) loses its linear character and it has not exact analytical solution for $T(t)$. Nevertheless, if the temperature dependence of effective heat transfer coefficient will be replaced by its dynamic character, i.e. by its time dependence in the heat transfer process, the equation (5) can be written as

$$\rho c_p L \frac{dT(t)}{T(t) - T_\infty} = -h(t)\,dt \tag{9}$$

with an analytical solution in the form of exponential temperature function of the generalized thermal lumped capacitance model

$$T(t) = T_\infty + \left(T_0 - T_\infty\right) exp\left[-\frac{1}{\rho c_p L}\int_0^t h(t)\,dt\right]. \tag{10}$$

In the temperature range where the dynamic character of effective heat transfer coefficient can be described by a linearly decreasing function of the time

$$h(t) = p_0 - p_1 t \tag{11}$$

with constant coefficients p_0 and p_1 [6], the temperature function (10) has a form of

$$T(t) = T_\infty + \left(T_0 - T_\infty\right) exp\left[-\frac{1}{\rho c_p L}\left(p_0 t - \frac{p_1 t^2}{2}\right)\right]. \tag{12}$$

At the same time, the coefficient p_0 represents an initial value of the effective heat transfer coefficient

$$p_0 = h(0) = h_c + \varepsilon\sigma \left(T_0 + T_\infty\right)\left(T_0^2 + T_\infty^2\right), \tag{13}$$

i.e. the value in the time instant t_0 at temperature T_0, while in

$$p_1 = \frac{p_0 - 4\varepsilon\sigma T_\infty^3}{t_f} \tag{14}$$

is its constant decay factor calculated for entire time of heat transfer process t_f, or in entire temperature interval $[T_0, T_\infty]$. However, the time window or temperature interval for the calculation of p_1 coefficient can have an optional length.
For an arbitrary polynomial time dependence of combined heat transfer coefficient with a decreasing trend the equation (10) goes to

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left[-\frac{1}{\rho c_p L}\int_0^t \left(p_0 - \sum_{i=1}^{n} p_i t^i\right) dt\right] \tag{15}$$

with exact analytical solution solution in the form of temperature functions

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left[-\frac{1}{\rho c_p L}\left(p_0 t - \sum_{i=1}^{n} \frac{p_i t^{i+1}}{i+1}\right)\right], \tag{16}$$

where n is the degree of plynomial.
The temperature functions of generalized lumped thermal capacitance model (16) allowe to predict analytically temperature of the solid at any instant of its coolig process, alternatively to determine the time needed to reach its required temperature. For $p_1 = 0$, i.e. in the case of constant heat transfer coefficients it has the analogical form as the temperature function of simplest lumped thermal capacitance model (1). It stands to reson that temperature functions (16) are able to describe analytically the cooled solid temperature-time history for a relatively wide range of temperature differences $T_0 - T_\infty$ [7]. Additionally, the analytical model (16) enables experimental estimation of combined heat transfer coefficient, e.g. by the application of sequential methods [8].

4. CONCLUSION

The described approach to solution of the total energy balance diffrential equation for the solid cooled by combined heat transfer through convection and radiation using the dynamic combined heat transfer coefficient representation, instead of its temperature dependence, makes it possible to obtain the analytical model for transient heat transfer analysis of cooling process of the solid through generalized lumped thermal capacitance approach.

5. REFERENCES

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